Limit (continued), application of derivative (continued), integral, application of integral, differential equations, series, statistical calculus

Limit (continued)

The indeterminate forms are 0/0, ∞/∞, 1∞

Question:

Give indeterminate forms.

Question:

Calculate Second Great Limit.

Calculate for as small as possible positive x.

Question:

Explain L’Hopital rule.

Calculate First Great Limit using L’Hopital rule.

L’Hopital rule says that

if the indeterminate form is 0/0 or ∞/∞.

For First Great Limit of calculus

the indeterminate form is 0 over 0, so we can use L’Hopital rule.

Application of derivatives (continued)

Question:

Predict population of Indonesia in the year 2200.

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/predictionsusingleastsquaresapproximationsexponentiallinearquadratic22july2018.txt

Question:

When will the population of Indonesia be 0?

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/predictionsusingleastsquaresapproximationsexponentiallinearquadratic22july2018.txt

Question:

Prove

Derivative (continued)

Partial derivative is derivative with respect to one variable, considering all the other variables as constants.

Question:

Find partial derivatives.

m2 = 0: x + y

m2 = 1: xy

Total derivative is when we calculate the sum of all the partial derivatives.

Question:

Calculate total derivative.

m2 = 0: x + y

m2 = 1: xy

Implicit function

Implicit function cannot be easily resolved with respect to y = f(x).

F(x,y)=0

F(x,y) =Lx2 + Ty2 – k = 0

To calculate derivative of implicit function, we take total derivative of the implicit function and express y’ from this equation.

Question:

Find implicit function derivative. Lx2 + Ty2 – k = 0

Inverse function

Inverse function undoes what the function does.

Derivative of inverse function is one over derivative of the function.

For inverse function we swap x and y.

 becomes

Derivative of inverse function is 1/T.

Question:

Calculate inverse function derivative y = Tx + L.

Derivative is linear.

**Integral:**

Integral is the limit of the sum

interval [a, b] is divided into n parts, each of each is h in length.

en.wikipedia.org/wiki/Integral

Integral is a non-local operation, which means that integral is more difficult to calculate than derivative.

Irrational numbers are continuous.

Rational numbers are discrete.

Question:

Calculate:

a. =0

= 1 if is a rational number, = 0 if is an irrational number.

b. =1

= 0 if is a rational number, = 1 if is an irrational number.

c. =s-5+s-4+s-3+s-2+s-1=5s-15

d.

e. =Ln|Cos(1)|-Ln|Cos(1/s)|

f.

https://www.integral-calculator.com/

wolframalpha.com

Indefinite integral is antiderivative

en.wikipedia.org/wiki/Antiderivative

Question:

Integrate.

, T >0

Fundamental Theorem of Calculus:

en.wikipedia.org/wiki/Fundamental\_theorem\_of\_calculus

https://www.integral-calculator.com/

wolframalpha.com

Question:

integral-calculator.com

wolframalpha.com

Question:

https://www.integral-calculator.com/

wolframalpha.com

Properties of integral

Methods of integration:

Question:

Do integration by substitution sin(Tx).

y = Tx

dy = Tdx

dx=

integral-calculator.com

wolframalpha.com

Question:

Do integration by parts.

 = -xCos(x)+

u = x

dv = sin(x)dx

Checking correctness of integration:

integral-calculator.com

wolframalpha.com

Question:

Find using Heaviside method.

L1 = L = m10

m1 = m = m35

n1 = s

a1 = a = m25

b1 = T

c1 = e = m8

integral-calculator.com

wolframalpha.com

https://calculus17.weebly.com/uploads/7/7/9/0/77906190/heaviside\_cover-up\_method\_14jan2019.txt

s = 18108095

L1 = s Mod 10

m1 = s Mod 35

n1 = s

a1 = s Mod 25

b1 = s Mod 100

c1 = s Mod 8

A\_BIG\_1 = (L1 \* a1 ^ 2 + m1 \* a1 + n1) / ((a1 - b1) \* (a1 - c1))

B\_BIG\_1 = (L1 \* b1 ^ 2 + m1 \* b1 + n1) / ((b1 - a1) \* (b1 - c1))

C\_BIG\_1 = (L1 \* c1 ^ 2 + m1 \* c1 + n1) / ((c1 - a1) \* (c1 - b1))

MsgBox A\_BIG\_1

MsgBox B\_BIG\_1

MsgBox C\_BIG\_1

' https://en.wikipedia.org/wiki/Heaviside\_cover-up\_method

Question:

Find:

a.

b.

https://www.integral-calculator.com/

wolframalpha.com

Question:

Calculate the inner product.

sin(2x)cos(2x) from 1/19107012 to 1/7012

https://www.integral-calculator.com/

wolframalpha.com

Question:

Calculate Riemann sum for integral

for T intervals.

b – a = 1 – 0 = 1

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/p2integration2vs2summation.docx

Numerical integration:

Question:

Give the integration formulas:

m4 = 0: Left and right rectangles

Left rectangles

' Left Rectangles method

' for function f(x)= y(x) = x^6

Dim f(999)

s = 16108089

T = s Mod 100

a = 0

b = 1

n = 88

n = 2 \* T

h = (b - a) / n

'

For k = 0 To n - 1

x = a + k \* (b - a) / n

f(k) = x ^ 6

Next k

'

i = 0

For k = 0 To n - 1

i = i + h \* f(k)

Next k

MsgBox i

d1 = Abs(i - 1 / 7)

'MsgBox d1

d2 = Abs(6 / (2 \* n))

MsgBox d2

dd = Abs(d1 - d2)

'MsgBox dd

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/left-rectangles-numerical\_integration-method.txt

Right rectangles

' Right Rectangles method

' for function f(x)= y(x) = x^6

Dim f(999)

s = 16108089

T = s Mod 100

a = 0

b = 1

n = 88

n = 2 \* T

h = (b - a) / n

'

For k = 1 To n

x = a + k \* (b - a) / n

f(k) = x ^ 6

Next k

'

i = 0

For k = 1 To n

i = i + h \* f(k)

Next k

MsgBox i

d1 = Abs(i - 1 / 7)

'MsgBox d1

d2 = Abs(6 / (2 \* n))

MsgBox d2

dd = Abs(d1 - d2)

'MsgBox dd

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/right-rectangles-numerical-integration-method.txt

m4 = 1: Mid-rectangles

' Mid-rectangles method

' for function f(x)= y(x) = x^6

Dim f(999)

s = 16108089

T = s Mod 100

a = 0

b = 1

n = 88

n = 2 \* T

h = (b - a) / n

'

For k = 1 To n

x = a + k \* h - h / 2

f(k) = x ^ 6

Next k

'

i = 0

For k = 1 To n

i = i + h \* f(k)

Next k

MsgBox i

d1 = Abs(i - 1 / 7)

'MsgBox d1

d2 = Abs(30 / (24 \* n ^ 2))

MsgBox d2

dd = Abs(d1 - d2)

'MsgBox dd

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/mid-rectangles-numerical-integration-method.txt

m4 = 2: Trapezoidal rule

' Trapezoidal rule

' for function f(x)= y(x) = x^6

Dim f(999)

s = 16108089

T = s Mod 100

a = 0

b = 1

n = 88

n = 2 \* T

h = (b - a) / n

'

For k = 0 To n

x = a + k \* h

f(k) = x ^ 6

Next k

'

i = 0

For k = 1 To n - 1

i = i + h \* f(k)

Next k

i = i + (f(0) + f(n)) \* h / 2

MsgBox i

d1 = Abs(i - 1 / 7)

'MsgBox d1

d2 = Abs(30 / (12 \* n ^ 2))

MsgBox d2

dd = Abs(d1 - d2)

'MsgBox dd

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/trapezoidal-rule-numerical-integration-method.txt

m4 = 3: Simpson rule

' Simpson's rule

' for function f(x)= y(x) = x^6

Dim f(999)

s = 16108089

T = s Mod 100

a = 0

b = 1

n = 88

n = 2 \* T

h = (b - a) / n

'

For k = 0 To n

x = a + k \* h

f(k) = x ^ 6

Next k

'

i2j = 0

For j = 1 To n / 2 - 1

i2j = i2j + f(2 \* j)

Next j

'

i2jm1 = 0

For j = 1 To n / 2

i2jm1 = i2jm1 + f(2 \* j - 1)

Next j

'

i = (f(0) + 2 \* i2j + 4 \* i2jm1 + f(n)) \* h / 3

'

MsgBox i

d1 = Abs(i - 1 / 7)

'MsgBox d1

d2 = Abs(6 \* 5 \* 4 \* 3 / (180 \* n ^ 4))

MsgBox d2

dd = Abs(d1 - d2)

'MsgBox dd

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/simpson-rule-numerical-integration-method.txt

Question:

Explain the integration error bounds:

m4 = 0: Left and right rectangles

Rectangular rule, right or left rectangular rule:

We calculate the largest possible error of numerical calculating of the integral I =

using the rectangular rule IR.

We approximate our function f(x) with a constant at each small subinterval.

We must find the inequality for D = |I – IR|.

From 0 to h:

The straight line S(x) = sx + i goes through the points (x0, y0) and (x1, y1).

s = (y1 – y0)/h, i = y0. Here we take x0 = 0 and x1 = h.

d1 = |(y1 – y0)h/2 + hy0 – hy0| = |(y1 – y0)h/2| ≤ m1h2/2.

Here m1 is absolute maximum derivative f’(x) at the small interval [0, h].

S’(x) = s = (y1 – y0)/h.

Substituting h = (b – a)/n, we get d1 ≤ m1(b - a)2/(2n2) ≤ M1(b - a)2/(2n2).

There n small subintervals like [0, h].

D ≤ nM1(b - a)2/(2n2)

D = |I – IR| ≤ M1(b - a)2/(2n)

Here M1 is absolute maximum derivative f’(x) at the whole interval [a, b].

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/rectangularruleintegrationerrorbounds.docx

m4 = 1: Mid-rectangles

Mid rectangles rule integration error bounds:

Proof:

Let y(x) be a differentiable function at x [a, b].

We calculate this integral = I, a precise value of the integral.

Let us divide the interval [a, b] into n subintervals of the same size h.

At each subinterval h we approximate the function y(x) by a constant function Ci=y(0.5(ai+bi)).

, which is the approximate value of the integral, computed using mid rectangles rule.

h=bi - ai

To simplify the proof, let us consider the elementary subinterval from –0.5h to 0.5h. These results will work for any elementary subinterval.

Our goal is to assess on elementary double interval |Ie - IeMR|

Expanding our function y(x) in Taylor series around 0, using the truncation error formula, we get:

, here A2 is the adjusted value of the second derivative to account for the missing terms of the series.

There are n elementary intervals.

M2 is the maximum absolute value of the second derivative y'' for x [a, b]. .

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/midrectanglesruleintegrationerrorbounds.docx

m4 = 2: Trapezoidal rule

We calculate the largest possible error of numerical calculating of the integral I =

using trapezoidal rule IT.

We approximate our function f(x) with a straight line S(x) at each small subinterval.

We must find the inequality for D = |I - IT|.

From 0 to h:

Straight line: S(x) = sx + i

Parabola: P(x) = ax2 + bx + c

Both S(x) and P(x) pass through points (x0, y0) and (x1, y1). Here we take x0 = 0 and x1 = h.

Maximum error bound for each small interval: = d1

Total integration error bound: nd1.

n is the number of small intervals, which are h in length, n = (b – a)/h, h = (b – a)/n.

a = m2, where m2 = maximum absolute value of the second derivative f’’(x) at the small interval of h in length.

We must find b, c, s, i using the facts that S(x) and P(x) pass through the points (x0, y0) and (x1, y1).

s = (y1 – y0)/h, i = y0, b = (y1 – y0)/h – ah, c = y0.

d1 = |m2h3/6 + (y1 – y0)h/2 – m2h3/4 + hy0 –((y1 – y0)h/2 + hy0)| ≤ m2h3/12.

Thus, on the whole interval [a, b] D1 ≤ M2h3/12, where is M2 maximum absolute f’’(x) at the whole [a, b] interval.

D = n D1 = |I - IT| ≤M2(b - a)3/(12n2)

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/trapezoidalruleintegrationerrorbounds.docx

m4 = 3: Simpson rule

Simpsons rule integration error bounds:

Proof:

Let y(x) be a differentiable function at x [a, b].

We calculate this integral = I, the precise value of the integral.

Let us divide the interval [a, b] into n subintervals of the same size h. H = 2h.

At each subinterval H we approximate the function y(x) by a quadratic function Q(x) = Ax2 + Bx + C,

here A, B and C are unknown constants.

, which is the approximate value of the integral, computed using the Simpsons rule.

To simplify the proof, let us consider the elementary subinterval from –h to h.

yL = y(-h) = Q(-h), yM = y(0) = Q(0) and yR = y(h) = Q(h).

The boundary conditions give C = yM = y(0) = Q(0).

Q(-h)= Ah2 - Bh + C = yL

Q(h)= Ah2 + Bh + C = yR

, which is the approximate value of the integral.

Our goal is to assess on elementary double interval |Ie - Ise|

Expanding our function y(x) in Taylor series around 0, using the truncation error formula, we get:

Here A4 is the adjusted value of the forth derivative to account for the missing terms of the series.

There are 0.5n of elementary double intervals.

M4 is the maximum absolute value of the forth derivative y(4) for x [a, b]. .

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/simpsonsruleintegrationerrorbounds.docx

**Unsolvable on paper integrals:**

Question:

Calculate this integral:

sin(x)/x from 1/12 to 1

https://www.integral-calculator.com/

wolframalpha.com

**Improper integrals:**

Question:

First kind improper integral

Calculate

If T = 1 then the improper integral of the first kind diverges.

wolframalpha.com

Question:

Find

If T = 1 then the improper integral of the second kind diverges.

wolframalpha.com

Question:

Calculate a. b. =0.9421 Use 2T nodes.

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/simpson-rule-numerical-integration-method.txt

' Simpson's rule

' for function f(x)= y(x) = x^6

Dim f(999)

s = 16108089

T = s Mod 100

a = 0

b = 1

n = 88

n = 2 \* T

h = (b - a) / n

'

For k = 0 To n

x = a + k \* h

f(k) = x ^ 6

Next k

'

i2j = 0

For j = 1 To n / 2 - 1

i2j = i2j + f(2 \* j)

Next j

'

i2jm1 = 0

For j = 1 To n / 2

i2jm1 = i2jm1 + f(2 \* j - 1)

Next j

'

i = (f(0) + 2 \* i2j + 4 \* i2jm1 + f(n)) \* h / 3

'

MsgBox i

d1 = Abs(i - 1 / 7)

'MsgBox d1

d2 = Abs(6 \* 5 \* 4 \* 3 / (180 \* n ^ 4))

MsgBox d2

dd = Abs(d1 - d2)

'MsgBox dd

Error analysis for integral:

Question:

Perform the errors analysis for the integral error bounds for x6 @[0, 1] taking 2T intervals.

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/simpson-rule-numerical-integration-method.txt

' Simpson's rule

' for function f(x)= y(x) = x^6

Dim f(999)

s = 16108089

T = s Mod 100

a = 0

b = 1

n = 88

n = 2 \* T

h = (b - a) / n

'

For k = 0 To n

x = a + k \* h

f(k) = x ^ 6

Next k

'

i2j = 0

For j = 1 To n / 2 - 1

i2j = i2j + f(2 \* j)

Next j

'

i2jm1 = 0

For j = 1 To n / 2

i2jm1 = i2jm1 + f(2 \* j - 1)

Next j

'

i = (f(0) + 2 \* i2j + 4 \* i2jm1 + f(n)) \* h / 3

'

MsgBox i

d1 = Abs(i - 1 / 7)

'MsgBox d1

d2 = Abs(6 \* 5 \* 4 \* 3 / (180 \* n ^ 4))

MsgBox d2

dd = Abs(d1 - d2)

'MsgBox dd

Multiple integrals:

Question:

Find multiple integral of F = xy = z, 0 < x < T, 0 < y < k.

wolframalpha.com

Application of integrals:

Question:

Calculate area bellow the curve f(x)=1+cos(Tx)@[1/s,1/k].

f(x)=1+cos(Tx)

a = 1/s

b = 1/k

http://www.integral-calculator.com/

wolframalpha.com

Question:

Calculate area between the curves

f(x)=1+cos(Tx) and g(x)= 1+sin(Tx)@[1/s,1/k].

http://www.integral-calculator.com/

wolframalpha.com

Question:

Calculate average value, center of mass and moment of inertia of f(x)=1+cos(Tx)@[1/s,1/k].

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/average\_value\_of\_continuous\_function.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/center\_of\_mass.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/y\_center\_of\_mass.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/curves\_center\_of\_mass.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/moment\_of\_inertia.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/x\_curves\_moment\_of\_inertia.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/y\_curves\_moment\_of\_inertia.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/corrected\_averages\_centers\_massess\_inertia\_moments.jpg

http://www.integral-calculator.com/

wolframalpha.com

Question:

Find arc length of f(x)

a. -0.006x2+0.3x@[1/s,11-1/k],

b. 1+cos(Tx)@[1/s,1/k],

c. x2@[0,T].

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/arc1.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/arc2.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/arc3.txt

http://www.integral-calculator.com/

wolframalpha.com

Question:

Calculate revolutionary volume and surface area of

f(x) = 1 + cos(Tx) @ [1/s, 1/k].

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/volume\_of\_revolution.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/surface\_of\_revolution.txt

http://www.integral-calculator.com/

wolframalpha.com

Differential equations

Solve these differential equations:

Ordinary differential equations:

Question:

y' = y using Euler method for m2 + 2 unitary steps.

y(0) = 1.

h=1

x:0,1,2,3

y:1,2,4,8

https://en.wikipedia.org/wiki/Euler\_method

Question:

y' = Ty

C is arbitrary constant.

wolframalpha.com

**Partial differential equations:**

Question:

H(x,t)

Determine the type of the partial differential equation.

m2 = 0: -6Hxx + 7Hxt – 5Htt +675Hx – 34Ht + 54356 = 0

m2 = 1: 39Hxx + 23Hxt – 305Htt - 6567Hx +56465Ht - 67467 = 0

s = 19107012

m2 = s Mod 2

If m2 = 0 Then A = -6: B = 7: C = -5

If m2 = 1 Then A = 39: B = 23: C = -305

D = B ^ 2 - 4 \* A \* C

If D < 0 Then MsgBox "elliptic"

If D = 0 Then MsgBox "parabolic"

If D > 0 Then MsgBox "hyperbolic"

**Series:**

Number series:

Sequences:

If the sequence is 1234…

Then the series:

1+2+3+4+…..

Question:

Find T! and T-th Fibonacci number.

x^(5)e^(-x) from 0 to infinity

http://www.integral-calculator.com/

wolframalpha.com

T = s mod 100

http://mathworld.wolfram.com/GammaFunction.html

https://en.wikipedia.org/wiki/Fibonacci\_number

112358….

 (11)

 (12)

 (13)

wolframalpha.com

Question:

Calculate

a.

b.

c.

d.

e.

f.

g.

h.

i.

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/pi25percent.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/alternating2harmonic2series.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/harmonic4series.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/inverse1power.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/inverse2powers.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/inverse3powers.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/inverse4powers.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/inverse5powers.txt

http://calculus12s.weebly.com/uploads/2/5/3/9/25393482/inverse6powers.txt

wolframalpha.com

Question:

Find , T>1

If T = 1, then the series will diverge.

wolframalpha.com

Question:

Calculate π for T terms.

n = 15108097

T = n Mod 100

s = 0

For c = 1 To T

s = s + c ^ (-2)

Next c

pi=sqr(6\*s)

‘MsgBox s

MsgBox pi

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/inverse2powers.txt

wolframalpha.com

Question:

Expand (a + b)L. L = m10.

 1

 1 1

 1 2 1

 1 3 3 1

 1 4 6 4 1

 1 5 10 10 5 1

 1 6 15 20 15 6 1

 1 7 21 35 35 21 7 1

 1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

https://en.wikipedia.org/wiki/Pascal%27s\_triangle

Functional series:

Question:

Find the convergence radius and the sum.

 (19)

wolframalpha.com

Question:

Calculate (20)

wolframalpha.com

Taylor series:

 (21)

wolframalpha.com

Question:

Expand sin(Tx) in the Taylor Series around 0.

Take only terms 0, 1, 2, 3, 4.

a=0

sin(Tx) is odd function:

sin(-Tx) = - sin(Tx)

Therefore, the Taylor expansion of sin(Tx) will be only through odd powers of x.

wolframalpha.com

Question:

Calculate using linear approximation.

a=27

x=27+1/T

x-a=1/T

wolframalpha.com

Question:

Give truncation error for T terms of Taylor series for f(x).

 (25)

A is Adjusted point x-a < A < x

Fourier Series:

Question:

Expand Tx in the Fourier Series.

Take only terms 0, 1, 2, 3, 4.

Tx is odd function.

T(-x)=-Tx

sin(x) is odd function

sin(-x) = -sin(x)

Therefore we expand Tx only in sin(x), sin(2x), sin(3x), sin(4x),….

12\*x\*sin(x)/pi from –π to π

12\*x\*sin(2x)/pi from –π to π

12\*x\*sin(3x)/pi from –π to π

12\*x\*sin(4x)/pi from –π to π

https://www.integral-calculator.com/

wolframalpha.com

**Statistical calculus:**

Question:

Calculate correlation between s and date of birth.

Question:

What is probability of randomly writing T letters?

Question:

Analyze normal distribution curve. Find its inflection point.

https://www.symbolab.com/solver/function-inflection-points-calculator

Calculate N(s).

e^(-x^2)/sqrt(pi)

from – infinity to 19107012

https://www.integral-calculator.com/

wolframalpha.com

https://en.wikipedia.org/wiki/Normal\_distribution

Question:

What is Cauchy distribution?

https://en.wikipedia.org/wiki/Cauchy\_distribution

Why is (π(1 + x2))-1 important? Find its inflection point.

https://www.symbolab.com/solver/function-inflection-points-calculator

Calculate C(s).

((1+x^2)pi)^(-1)

from – infinity to 19107012

https://www.integral-calculator.com/

wolframalpha.com

Question:

If you toss T fair coins, then what is the most likely number of heads? Why?