Mid rectangles rule integration error bounds:

Proof:

Let y(x) be a differentiable function at x [a, b].

We calculate this integral = I, a precise value of the integral.

Let us divide the interval [a, b] into n subintervals of the same size h.

At each subinterval h we approximate the function y(x) by a constant function Ci=y(0.5(ai+bi)).

, which is the approximate value of the integral, computed using mid rectangles rule.

h=bi - ai

To simplify the proof, let us consider the elementary subinterval from –0.5h to 0.5h. These results will work for any elementary subinterval.

Our goal is to assess on elementary double interval |Ie - IeMR|

Expanding our function y(x) in Taylor series around 0, using the truncation error formula, we get:

, here A2 is the adjusted value of the second derivative to account for the missing terms of the series.

There are n elementary intervals.

M2 is the maximum absolute value of the second derivative y'' for x [a, b]. .