Notes for calculus UTS midterm exam:

Study materials:

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/e1calc2023.docx

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/e2calc2pres2023.docx

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/e3calc2023.docx

s is your student number.

k = s mod 10000

T = s mod 100

m = s mod 35

L = s mod 10

1. Solve |k - Tx| < s.

|k - Tx| < s

|Tx - k| < s

-s + k < Tx < s + k

http://www.wolframalpha.com

Question:

Classify shape Tx2 + mxy + Ly2 = 1.

s = 23123456

L = s Mod 10

m = s Mod 35

T = s Mod 100

A = T

B = m

C = L

D = B ^ 2 - 4 \* A \* C

If D < 0 Then MsgBox "ellipse"

If D = 0 Then MsgBox "parabola"

If D > 0 Then MsgBox "hyperbola"

2. Analyse exponential growth and decay.

For what x is eLx = 0.5?

s = 23123456

L = s Mod 10

x = Log(0.5) / L

MsgBox x

What gives the greater value 0.1T% decay in 2 years or 0.05T % every year? Why?

s = 23123456

T = s Mod 100

c2 = Sqr(1 – 0.1 \* T \* 0.01)

c1 = 1 + 0.05\*T \* 0.01

MsgBox c1 - c2

Calculate limit

symbolab.com/solver/limit-calculator

3. Find logistic function P(t) for i = L+1 and R = t = M = L+2.

s = 23123456

L = s Mod 10

i = L + 1

M\_BIG = L + 2

r = L + 2

t = L + 2

P = M\_BIG \* i \* Exp(r \* t) / (M\_BIG + i \* Exp(r \* t) - 1)

MsgBox P

4. Find discriminant of y2=x3+Lx+T.

D = -16(4L3 + 27T2)

s = 23123456

L = s Mod 10

T = s Mod 100

a = L

b = T

D = -16 \* (4 \* a \* a \* a + 27 \* b \* b)

MsgBox D

Question:

Apply limit, derivative, integral.

. f(x) = Tx + k. For any ε find δ, using ε – δ definition of the limit.

m4 = 0: Use L’Hopital rule to prove First Great Limit of Calculus: 1

m4 = 1: 1

m4 = 2:

m4 = 3:

symbolab.com/solver/limit-calculator

Find min and MAX.

MAX and min are at points where f’(x) = 0, these are called critical points.

If f’(x) < 0 then f’(x) > 0, then there is minimum.

If f’(x) > 0 then f’(x) < 0, then there is MAXIMUM.

Find the largest area rectangle with perimeter of T meters.

P is Perimeter.

A is Area.

x is width.

y is height.

P = 2x+2y (1)

A = xy (2)

A is minimum when x = 0 or y = 0, in these cases A = xy = 0.

From (1)

 (3)

We put y from equation (3) to equation (2)

 (5)

Differentiating or taking the derivative from (5), we get (6)

 (6)

x=0.25P

Derivative is positive at (-∞, )

If x < 0.25P, then then the derivative is positive:

Derivative is negative at ()

Therefore x = 0.25P is MAXIMUM.

x=0.25P (4)

Putting (4) into (3)

P = T.

T = s mod 100.

From (2)

Rectangle with the largest area is square, if perimeter is the same.

Calculate the largest area right-angled triangle with perimeter of T meters.

a = b

Find maximum volume cylinder for surface area of T meters square. T = 2πRH (5)

For maximum volume R = H.

Maximum volume is

Calculate maximum volume cone for surface area of T meters square. T = πRL

R = H

Calculate maximum area scalene triangle with perimeter of T meters.

All the sides of the triangle must be the same in length to achieve the largest area for the same perimeter.

Calculate:

a. =0

= 1 if is a rational number, = 0 if is an irrational number.

b. =1

= 0 if is a rational number, = 1 if is an irrational number.

, T >0

https://www.integral-calculator.com/

wolframalpha.com

Question:

integral-calculator.com

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Do integration by substitution sin(Tx).

y = Tx

dy = Tdx

dx=

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Question:

Do integration by parts.

 = -xCos(x)+

u = x

dv = sin(x)dx

Checking correctness of integration:

integral-calculator.com

wolframalpha.com

First kind improper integral

Calculate

If T = 1 then the improper integral of the first kind diverges.

wolframalpha.com

Question:

Find

If T = 1 then the improper integral of the second kind diverges.

wolframalpha.com

5. Prove expression for derivative of x2 using limit.

Question:

Find

6. Find linear fit for (2, m2), (3, m3), (4, m4).

Dim x(3), y(3)

s = 23123456

m = 3

x(1) = 2

x(2) = 3

x(3) = 4

y(1) = s mod 2

y(2) = s mod 3

y(3) = s mod 4

sx = 0

For j = 1 To m

sx = sx + x(j)

Next j

sy = 0

For j = 1 To m

sy = sy + y(j)

Next j

sxy = 0

For j = 1 To m

sxy = sxy + x(j) \* y(j)

Next j

sx2 = 0

For j = 1 To m

sx2 = sx2 + x(j) ^ 2

Next j

g = (m \* sxy - sx \* sy) / (m \* sx2 - sx ^ 2)

i = (sy - g \* sx) / m

MsgBox g

MsgBox i

7. Find implicit function derivative Lx2 + Ty2 – k = 0.

8. Find inverse function derivative for Tx + L.

Inverse function undoes what the function does.

Derivative of inverse function is one over derivative of the function.

For inverse function we swap x and y.

 becomes

inverse function of 14x + 4 is

x = 14y + 4,

Derivative of inverse function is 1/T.

Question:

Find curvature of Tx2+Lx.

y''=2T

y' = 2Tx+L

9. Solve differential equations:

Solve:

y' = y

y' = y using Euler method for m2 + 2 unitary steps.

y(0) = 1.

h=1

x:0,1,2,3

y:1,2,4,8

Solve:

y' = Ty.

C is arbitrary constant.

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Question:

Find T! and FT.

x^(56)e^(-x) from 0 to infinity

http://www.integral-calculator.com/

wolframalpha.com

112358….

 (11)

 (12)

 (13)

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10. Find (a+b)L

 1

 1 1

 1 2 1

 1 3 3 1

 1 4 6 4 1

 1 5 10 10 5 1

 1 6 15 20 15 6 1

 1 7 21 35 35 21 7 1

 1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

Question:

Find

, T>1

If T = 1, then the series will diverge.

Question:

Find

11. Find

Calculate using linear approximation.

a=27

x=27+1/T

x-a=1/T

Question:

Find error for T terms of Taylor series.

Give truncation error for T terms of Taylor series for f(x).

 (25)

A is Adjusted point x-a < A < x

12. Expand sin(Tx) in Taylor Series.

Expand sin(Tx) in the Taylor Series around 0.

Take only terms 0, 1, 2, 3, 4.

a=0

sin(Tx) is odd function:

sin(-Tx) = - sin(Tx)

Therefore, the Taylor expansion of sin(Tx) will be only through odd powers of x.

Question:

Expand Tx in Fourier Series.

Expand Tx in the Fourier Series.

Take only terms 0, 1, 2, 3, 4.

Tx is odd function.

T(-x)=-Tx

sin(x) is odd function

sin(-x) = -sin(x)

Therefore we expand Tx only in sin(x), sin(2x), sin(3x), sin(4x),….

56\*x\*sin(x)/pi from –π to π

56\*x\*sin(2x)/pi from –π to π

56\*x\*sin(3x)/pi from –π to π

56\*x\*sin(4x)/pi from –π to π

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