Simpsons rule integration error bounds:

Proof:

Let y(x) be a differentiable function at x [a, b].

We calculate this integral = I, the precise value of the integral.

Let us divide the interval [a, b] into n subintervals of the same size h. H = 2h.

At each subinterval H we approximate the function y(x) by a quadratic function Q(x) = Ax2 + Bx + C,

here A, B and C are unknown constants.

, which is the approximate value of the integral, computed using the Simpsons rule.

To simplify the proof, let us consider the elementary subinterval from –h to h.

yL = y(-h) = Q(-h), yM = y(0) = Q(0) and yR = y(h) = Q(h).

The boundary conditions give C = yM = y(0) = Q(0).

Q(-h)= Ah2 - Bh + C = yL

Q(h)= Ah2 + Bh + C = yR

, which is the approximate value of the integral.

Our goal is to assess on elementary double interval |Ie - Ise|

Expanding our function y(x) in Taylor series around 0, using the truncation error formula, we get:

Here A4 is the adjusted value of the forth derivative to account for the missing terms of the series.

There are 0.5n of elementary double intervals.

M4 is the maximum absolute value of the forth derivative y(4) for x [a, b]. .